Objectives:
* Classify triangles and find the measures of their angles.
* Use exterior angles of triangles.
Check Skills You'll Need

Classify each angle as acute, right, or obtuse.

1. Right

2. Acute

3. Obtuse

Solve each equation.

4. \(30 + 90 + x = 180\)
   \(120 + x = 180\)
   \(x = 60\)

5. \(55 + x + 105 = 180\)
   \(x + 160 = 180\)
   \(x = 20\)

6. \(x + 58 = 90\)
   \(x = 32\)

7. \(32 + x = 90\)
   \(x = 58\)
1. **Activity**

- Draw and cut out a large triangle.
- Number the angles and tear them off.
- Place the three angles adjacent to each other to form one angle, as shown at the right.

1. What kind of angle is formed by the three smaller angles? What is its measure?

2. Make a conjecture about the sum of the measures of the angles of a triangle.
Theorem 3-12  Triangle Angle-Sum Theorem

The sum of the measures of the angles of a triangle is 180.

\[ m\angle A + m\angle B + m\angle C = 180 \]
**Proof of Theorem 3-12**

**Given:** \( \triangle ABC \)

**Prove:** \( m\angle A + m\angle B + m\angle 3 = 180 \)

**Proof:** By the Protractor Postulate, you can draw \( \overrightarrow{CP} \) so that \( m\angle 1 = m\angle A \). Then, \( \angle 1 \) and \( \angle A \) are congruent alternate interior angles, so \( \overrightarrow{CP} \parallel \overrightarrow{AB} \).

\( \angle 2 \) and \( \angle B \) are also alternate interior angles, so by the Alternate Interior Angles Theorem, \( m\angle 2 = m\angle B \).

By substitution, \( m\angle A + m\angle B + m\angle 3 = m\angle 1 + m\angle 2 + m\angle 3 \), which is equal to 180 by the Angle Addition Postulate.
### Example: Applying the Triangle Angle-Sum Theorem

**Algebra** Find the values of \( x \) and \( y \).

To find the value of \( x \), use \( \triangle GFJ \).

\[
39 + 65 + x = 180 \quad \text{Triangle Angle-Sum Theorem}
\]

\[
104 + x = 180 \quad \text{Simplify.}
\]

\[
x = 76 \quad \text{Subtract 104 from each side.}
\]

To find the value of \( y \), look at \( \angle FJH \). It is a straight angle.

\[
m\angle GJF + m\angle GJH = 180 \quad \text{Angle Addition Postulate}
\]

\[
x + y = 180 \quad \text{Substitute.}
\]

\[
76 + y = 180 \quad \text{Substitute 76 for } x.
\]

\[
y = 104 \quad \text{Subtract 76 from each side.}
\]

Find the value of \( z \) in two different ways, each way using the Triangle Angle-Sum Theorem.

\[
21 + 39 + 65 + z = 180
\]

\[
125 + z = 180
\]

\[
z = 55
\]
Classify each triangle by its angles.

- **Equiangular**: all angles congruent
- **Acute**: all angles acute
- **Right**: one right angle
- **Obtuse**: one obtuse angle

Classify each triangle by its sides.

- **Equilateral**: all sides congruent
- **Isosceles**: at least two sides congruent
- **Scalene**: no sides congruent
Example

Classifying a Triangle

Classify the triangle by its sides and its angles.

**ISOSCELES**

**ACUTE**

At least two sides are congruent, so the triangle is isosceles. All the angles are acute, so the triangle is acute.

- The triangle is an acute isosceles triangle.
An **exterior angle of a polygon** is an angle formed by a side and an extension of an adjacent side. For each exterior angle of a triangle, the two nonadjacent interior angles are its **remote interior angles**.

The diagram at the right suggests a relationship between an exterior angle and its two remote interior angles. Theorem 3-13 states this relationship. You will prove this theorem in Exercise 35.
Theorem 3-13  Triangle Exterior Angle Theorem

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

\[ m\angle 1 = m\angle 2 + m\angle 3 \]
3.4 Parallel Lines and the Triangle Angle Sum Theorem

EXAMPLE Using the Exterior Angle Theorem

**Algebra** Find each missing angle measure.

a.\[
\begin{align*}
\angle 1 &= 40^\circ + 30^\circ \\
\angle 1 &= 70^\circ
\end{align*}
\]

b.\[
\begin{align*}
113^\circ &= 70^\circ + \angle 2 \\
43^\circ &= \angle 2
\end{align*}
\]

3. Two angles of a triangle measure 45. Find the measure of an exterior angle at each vertex.
Multiple Choice  The lounge chair has different settings that change the angles formed by its parts. Suppose \( m \angle 2 \) is 32 and \( m \angle 3 \) is 81. Find \( m \angle 1 \), the angle formed by the back of the chair and the arm rest.

- A 67
- B 81
- C 113
- D 180

\[
m \angle 1 = m \angle 2 + m \angle 3
\]

Exterior Angle Theorem

Substitute.

Simplify.

The angle formed is a 113° angle.
The correct choice is C.
Homework:
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